Journal of Applied Mechanics and Technical Physics, Vol. 44, No. 3, pp. 400–405, 2003

EXPERIMENTAL DETERMINATION OF HYDRAULIC CONDUCTIVITY OF PINE SAMPLES IN THE LONGITUDINAL DIRECTION DURING CONVECTIVE DRYING

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UDC 532.72; 669.015.23

In studying the process of drying wood samples, the hydraulic conductivity of pine samples exposed to a convective air flow was measured in the direction perpendicular to the cross-sectional plane of the tree (along the wood grain). The kinetic curves of the drying process were treated with a technique based on the approximate solution of the one-dimensional diffusion equation for hydraulic conductivity of wood with a boundary condition of the third kind. The technique was tested on the basis of the known value of hydraulic conductivity in the direction tangential to annual rings of the tree. It is shown that the hydraulic conductivity in the longitudinal direction is 17 times the hydraulic conductivity in the direction tangential to the annual rings in the cross-sectional plane of the tree. By means of numerical simulation of the process, based on solving the initial-boundary problem for the two-dimensional linear equation of hydraulic conductivity, the effect of anisotropy of hydraulic conductivity coefficients on the dependence of the mean humidity on time and on the local humidity distribution is studied.

Key words: *convective drying, hydraulic conductivity of wood, anisotropy, physical and mathematical simulation.*

Experimental Investigation of Drying of an Anisotropic Body. It is known that wood has a rather complicated capillary-porous structure, which is anisotropic, i.e., different in three orthogonal directions: normal to the annual rings in the cross-sectional plane of the tree, tangential to the annual rings, and perpendicular to the cross-sectional plane [1]. Correspondingly, hydraulic conductivity of wood, which affects the rate of drying, is also anisotropic, i.e., depends on the chosen direction.

The data currently available in the literature contain information on hydraulic conductivity of wood in two directions: radial (perpendicular to the annual rings) and tangential to the annual rings. Concerning the third direction (along the wood grain), it is known that hydraulic conductivity in this direction is higher than that in the tangential direction by a factor of 15–20 [1]. It is of interest to measure the hydraulic conductivity of wood in the longitudinal direction and compare it with available data on hydraulic conductivity in other directions.

The pine samples were cut so that moisture transfer to their side surfaces was determined by longitudinal hydraulic conductivity. The typical size of the side surfaces of the boards (plates) was 40×40 mm, and the sample thickness was 3.05, 3.5, or 6.3 mm. The side surfaces of the samples coincided with the cross-sectional plane of the tree.

The samples were soaked during 30–40 min and seasoned in a closed thermostat during 24 hours to make the initial humidity profile uniform. Then, the pine samples were attached to a sting and exposed to an air flow produced by a fan at room temperature. The flow velocity was approximately 15 m/sec, and the air temperature was 22°C. With certain time intervals (15 min), the samples were weighted on a VLA-200-M analytical balance.

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Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 44, No. 3, pp. 117–123, May–June, 2003. Original article submitted November 25, 2002.

The experimental curves were treated by the technique described in [2, 3]. At the first stage of investigations, the influence of overflow along different directions was assumed to be weak.

The technique is based on the approximate solution of the one-dimensional equation of hydraulic conductivity for humidity of wood [3]

$$\frac{\partial W}{\partial t} = a\,\frac{\partial^2 W}{\partial x^2}$$

with the boundary condition of the third kind on the side surface of the sample

$$q = -a\rho_0\Big(\frac{\partial W}{\partial x}\Big)_{\rm s} = \alpha\rho_0(W_{\rm s} - W_{\rm eq}).$$

Here $W(x,t)$ is the local humidity equal to the ratio of the local mass of moisture to the local mass of the absolutely dry material, a is the hydraulic conductivity coefficient, q is the density of the moisture flux on the surface of the one-dimensional sample, ρ_0 is the density of the dry material, the subscript "s" indicates quantities calculated at points of the side surface of the sample, α is the moisture-transfer coefficient, and $W_{\rm eq}$ is the equilibrium (with the ambient medium) humidity.

For $t > 0.54R^2/a$, the approximate solution of the problem of sample drying, described by the third boundary-value problem for the hydraulic conductivity equation, has the form

$$\frac{W(x,t)}{\langle W_0\rangle} = 1 - \int\limits_0^t \frac{a\,{\rm Ki}(\tau)}{R^2}\,d\tau + {\rm Ki}(t)\,\frac{R^2 - 3x^2}{6R^2}.$$

Here R is the half-thickness of the plate, ${\rm Ki} = qR/(a\rho_0\langle W_0\rangle)$ is the Kirpichev criterion, and $\langle W_0\rangle$ is the mean initial humidity. The humidity averaged over the cross section is determined by the expression

$$\langle W\rangle = \frac{1}{R}\int\limits_0^R W(x,t)\,dx.$$

After some simple transformations, the equation for $\langle W\rangle$ becomes

$$-\frac{\langle W\rangle - W_{\rm p}}{R\,d\langle W\rangle/dt} = \frac{R}{3a} + \frac{1}{\alpha}. \qquad (1)$$

In the case of two-dimensional hydraulic conductivity in a beam, the problem is described by the diffusion equation with anisotropic hydraulic conductivity. The solution of the corresponding boundary-value problem was written in series [4]. After some simple transformations of this solution and its analysis, we can obtain the equation for the mean humidity:

$$\frac{\langle W\rangle - W_{\rm p}}{d\langle W\rangle/dt} = -\frac{R_1R_2(3a_1 + \alpha R_1)(3a_2 + \alpha R_2)}{3\alpha[3a_1a_2(R_1 + R_2) + \alpha(a_1R_1^2 + a_2R_2^2)]}.$$

Here R_1 and R_2 are the half-thicknesses of the beam in the x and y directions, respectively. Note, as $R_2 \to \infty$, this equation reduces to the one-dimensional equation (1), which allows one to use Eq. (1) to describe the results of experiments on determining the anisotropic coefficient of hydraulic conductivity.

It follows from Eq. (1) that, if we plot the quantity in the left side of the equation for identical values of humidity but for samples of different thickness as the ordinate, we obtain a straight line that cuts off a segment $1/\alpha$ on this axis at an angle $1/(3a)$. Thus, we can obtain the coefficients of moisture transfer and hydraulic conductivity. Strictly speaking, this technique is applicable for humidities lower than the hygroscopicity limit (below 30%), when the hydraulic conductivity equation is valid.

The technique was tested with the known value of hydraulic conductivity in the direction tangential to the annual rings. The difference of the results obtained ($3.3 \cdot 10^{-6}$ cm^2/sec) from those given in the literature for pine sapwood [3] is less than 12%.

The above-described experiments were repeated four times. Figure 1 shows the results of one experiment for samples of different thickness ($m_{\rm eq}$ and $m - m_{\rm eq}$ are the current mass of the sample and its mass equilibrium with the ambient medium, respectively). Figure 2 shows the dependence of the left side of Eq. (1), denoted as B, on the plate half-thickness R [$B(R) = 15.79R + 153.91$]. The mean hydraulic conductivity in the longitudinal direction obtained in the experiments was $6.2 \cdot 10^{-5}$ cm^2/sec ($\langle W\rangle \simeq 20\%$), whereas the hydraulic conductivity of pine sapwood in the tangential direction is $3.7 \cdot 10^{-6}$ cm^2/sec at a temperature of 20°C [3]. Thus, it was shown that

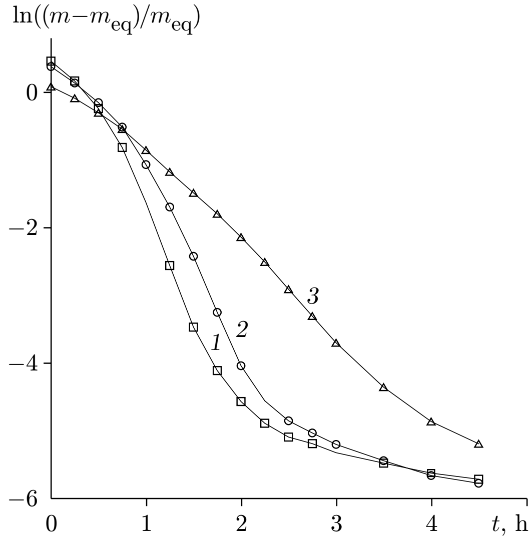

Fig. 1

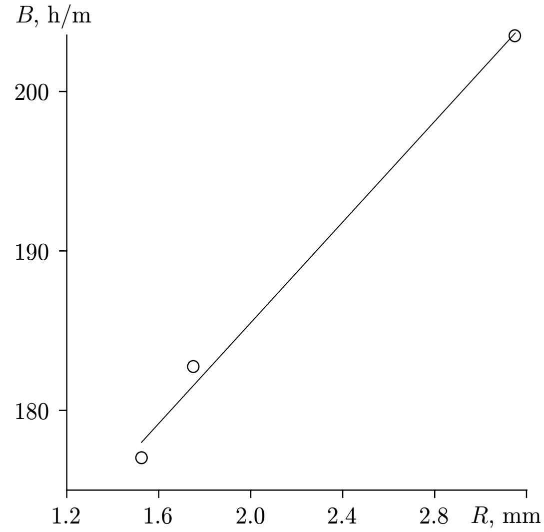

Fig. 2

Fig. 1. Drying rate of samples of different thickness: $2R = 3.0$ (1), 3.5 (2), and 6.3 mm (3).

Fig. 2. Dependence of the left side of Eq. (1) on the sample half-thickness: the points show the experimental data, and the curve is the approximating function.

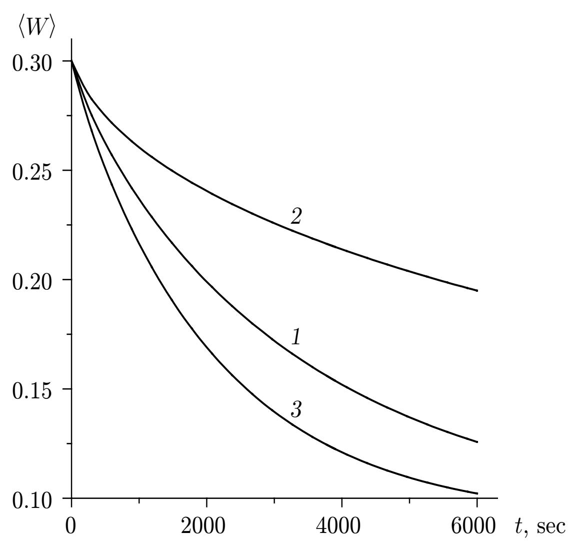

Fig. 3. Effect of anisotropy of hydraulic conductivity on the time evolution of the mean humidity in the sample: $a_1 = 6.2 \cdot 10^{-9}$ m^2/sec and $a_2 = 3.7 \cdot 10^{-10}$ m^2/sec (1), $a_1 = a_2 = 3.7 \times 10^{-10}$ m^2/sec (2), and $a_1 = a_2 = 6.2 \cdot 10^{-9}$ m^2/sec (3).

the hydraulic conductivity of pine wood in the longitudinal direction is almost 17 times the hydraulic conductivity in the tangential direction.

Mathematical Simulation of Moisture Extraction from an Anisotropic Sample. After experimental studies of hydraulic conductivity, it is of interest to evaluate the error in determining the fields of humidity by mathematical models that ignore anisotropy of physical properties of the sample. The effect of anisotropy of hydraulic conductivity on the kinetic curves of the drying process for two-dimensional samples was examined. The mathematical model describing moisture transfer in the sample during convective drying is a two-dimensional linear equation of hydraulic conductivity for samples with a square cross section (1×1 cm):

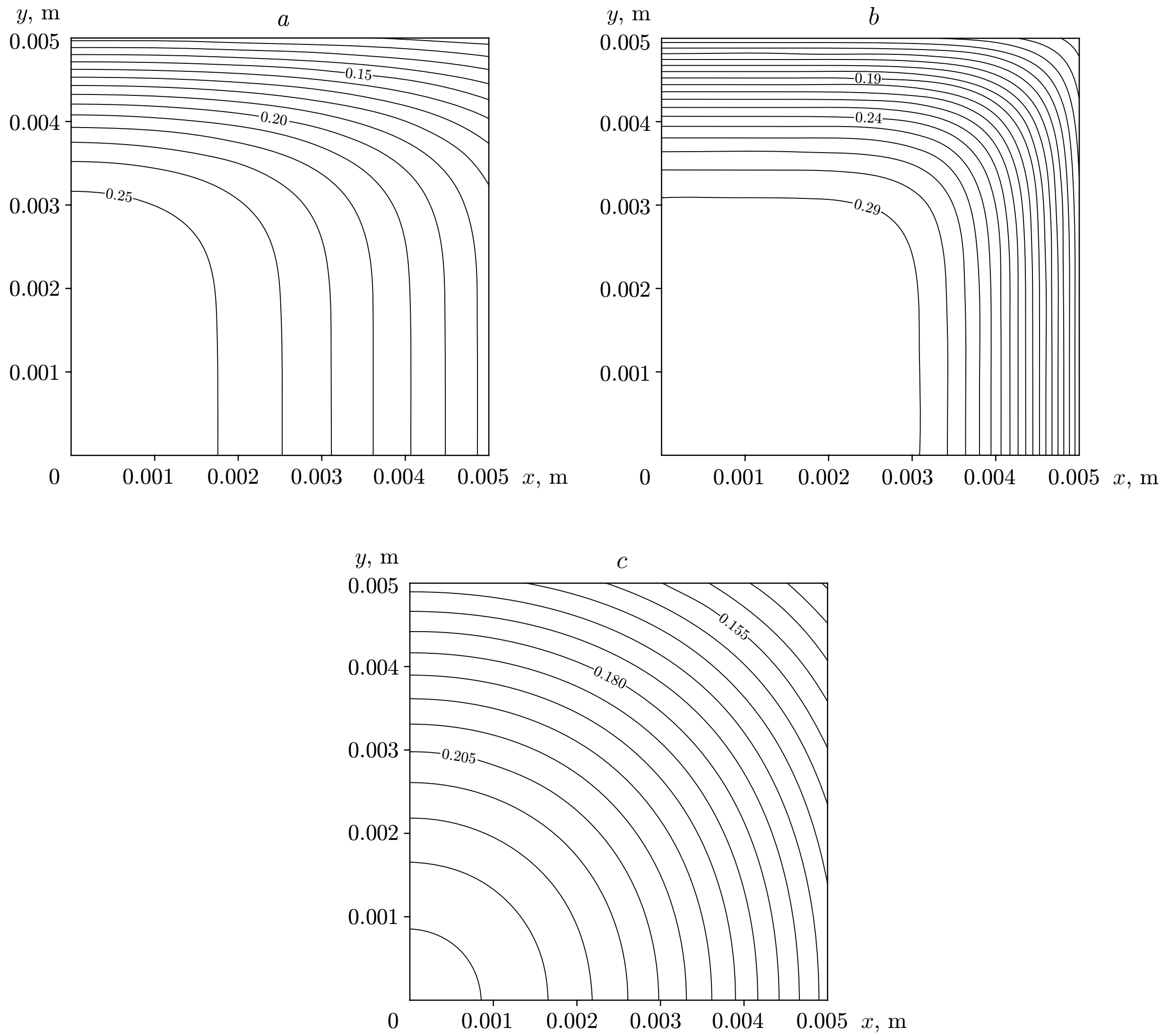

Fig. 4. Effect of anisotropy of hydraulic conductivity on the distribution of humidity over the sample ($\alpha = 1.76 \times 10^{-6}$ m/sec and $t = 1500$ sec): $a_1 = 6.2 \cdot 10^{-9}$ m^2/sec and $a_2 = 3.7 \cdot 10^{-10}$ m^2/sec (a), $a_1 = a_2 = 3.7 \cdot 10^{-10}$ m^2/sec (b), and $a_1 = a_2 = 6.2 \cdot 10^{-9}$ m^2/sec (c).

$$\frac{\partial W}{\partial t} = a_1 \frac{\partial^2 W}{\partial x^2} + a_2 \frac{\partial^2 W}{\partial y^2}.$$

The solution of this equation $W(x, y, t)$ should satisfy the boundary conditions of the third kind on the side surfaces of the sample

$$q_x = -a_1 \rho_0 \frac{\partial W}{\partial x}(R, y, t) = \alpha \rho_0 (W(R, y, t) - W_{\text{eq}}),$$

$$q_y = -a_2 \rho_0 \frac{\partial W}{\partial y}(x, R, t) = \alpha \rho_0 (W(x, R, t) - W_{\text{eq}}),$$

the condition of uniformity of the initial humidity in the sample

$$W(x, y, 0) = W_0,$$

and the condition of symmetry for zero values of abscissa and ordinate. Here q_x and q_y are the densities of moisture fluxes on the surfaces in the x and y directions, respectively.

The initial-boundary problem was solved by the method of lines. To test the method, calculations were performed on a sequence of condensing grids uniform in both directions. The results obtained with a spatial step

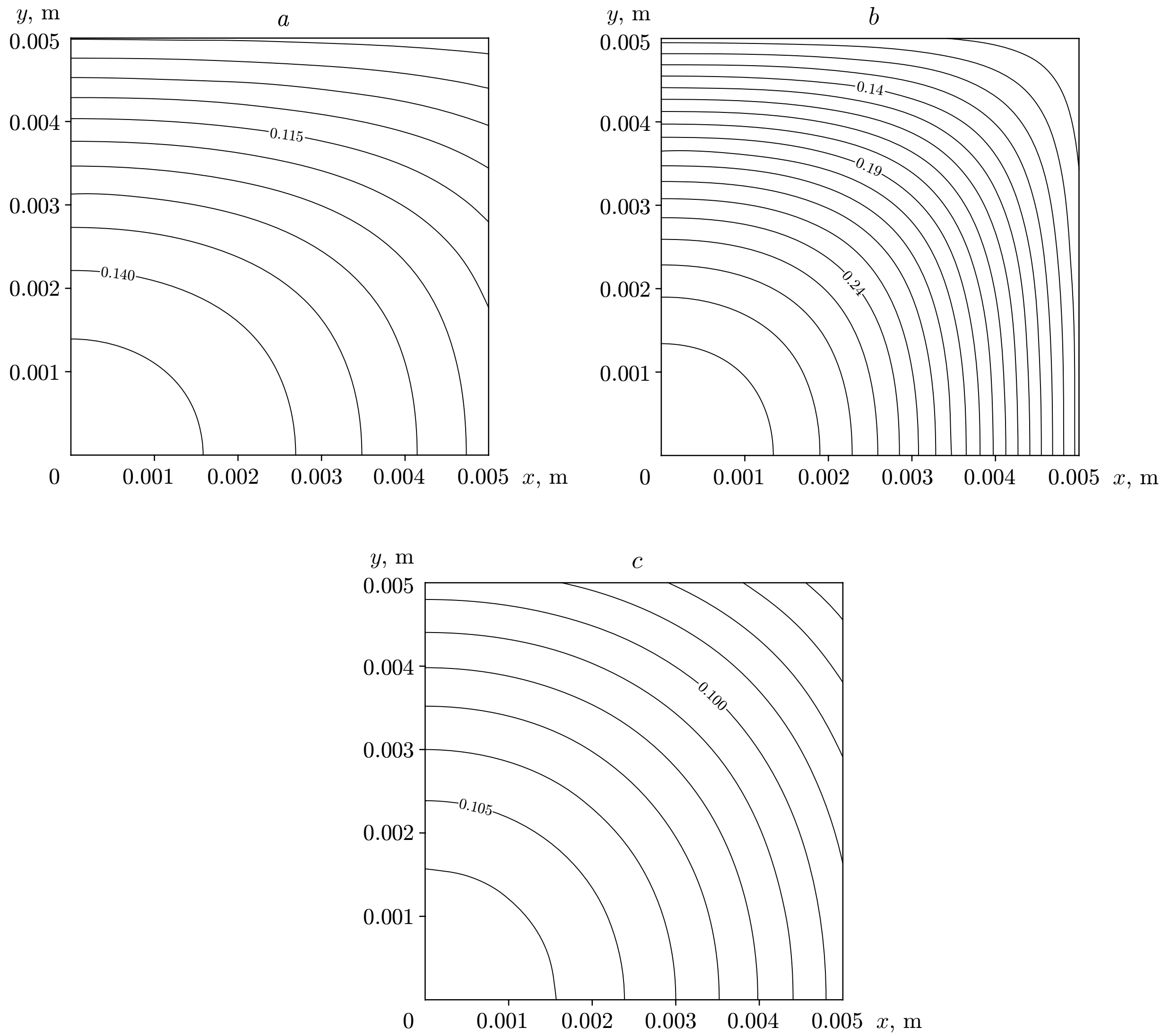

Fig. 5. Effect of anisotropy of hydraulic conductivity on the distribution of humidity over the sample ($\alpha = 1.76 \cdot 10^{-6}$ m/sec and $t = 6000$ sec) (notation the same as in Fig. 4).

$h = h_x = h_y$ and a step $h/2$ differ by less than 1% beginning from $h \approx 5 \cdot 10^{-5}$ m. Therefore, exactly this value of h was used in further calculations.

After the test calculations, three variants of initial data were considered. In the first variant of initial data, the hydraulic conductivities in two directions were different (the above-given experimental values of hydraulic conductivities in the longitudinal and tangential directions were used). In the second variant, the coefficients were identical and equal to the smaller of them. In the third variant, both coefficients were equal to the higher value of hydraulic conductivity. The calculation results are plotted in Fig. 3, which shows the dependences of the mean humidity determined by the formula

$$\langle W \rangle = \frac{1}{R^2} \int_0^R \int_0^R W(x, y, t)\, dx\, dy,$$

on time for three variants of initial data. The curve of the mean hydraulic conductivity calculated for the case of anisotropic hydraulic conductivity lies between the curves corresponding to the maximum and minimum coefficients of hydraulic conductivity.

Let us consider the dynamics of moisture transfer from the sample during convective drying. Figures 4 and 5 show the distributions of humidity over one quarter of the sample (due to spatial symmetry) for $t = 1500$ and

6000 sec. It follows from Figs. 4 and 5 that the effect of anisotropy of hydraulic conductivity on the humidity fields is quite significant. Indeed, the isolines of humidity in Fig. 4a are closer to each other along the ordinate axis than along the abscissa axis. This is caused by the influence of the boundary layer because of a rather low hydraulic conductivity along the y axis. For this variant, the process of drying in the x direction is more intense, leading to an asymmetric distribution of humidity. For the variants $a_1 = a_2$ plotted in Fig. 4b and c, a symmetric pattern of isolines is observed. In the case shown in Fig. 4b, the sample is dried slower than that in Fig. 4a. The isoline $W = 0.25$ in Fig. 4a is located closer to the sample center than that in Fig. 4b.

Figure 4a and b reveals a core with high humidity in the central part of the sample. The core becomes smaller as the hydraulic conductivity of the sample increases to the maximum value. This follows from a comparison of the positions of lines of constant humidity $W = 0.29$ (Fig. 4a and b). This isoline is absent in Fig. 4c. With increasing time of drying, this effect is attenuated because of the decrease in the integral humidity in the sample (cf. Figs. 4 and 5).

It should be noted that the effect of two-dimensionality of the process in drying boards is manifested at their ends in the region whose length is greater than the board thickness approximately by a factor of $\sqrt{17}$. Significant hydraulic conductivity in the longitudinal direction can lead to cracking at the board ends.

Conclusions. Thus, it is shown experimentally that the hydraulic conductivity coefficient in a wood sample is a tensor. Its value in the direction perpendicular to the cross-sectional plane of the tree (along the wood grain) in a plane sample is determined.

Within the framework of the linear mathematical model of moisture transfer in a wood sample, Lykov's formula is generalized to the case of plane anisotropic samples. A mathematical model is developed, which reveals the effect of anisotropy of hydraulic conductivity on the drying process.

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